Basics of Fluid Construction Grammar.

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Abstract

Fluid Construction Grammar (FCG) is a fully operational computational platform for developing grammars from a constructional perspective. It contains mechanisms for representing grammars and for using them in computational experiments and applications in language understanding, production and learning. FCG can be used by grammar writers who want to test whether their grammar fragments are complete and coherent for the domain they are investigating (for example verb phrases) or who are working in a team and have to share grammar fragments with others. It can be used by computational linguists implementing practical language processing systems or exploring how machine learning algorithms can acquire grammars. This paper introduces some of the basic mechanisms of FCG, illustrated with examples.

Keywords: Construction Grammar, Computational Construction Grammar, Fluid Construction Grammar.

1 Introduction

Despite the great interest and growing success of construction grammar in empirical linguistics, diachronic linguistics, language teaching, child language research, and other fields of language studies, there is still no widely accepted formalism for defining construction grammars nor a robust easily accessible computational implementation that operationalises how construction schemas can be used in parsing or producing utterances. Such a computational formalism would be useful in order to test grammar fragments against a corpus of example cases. Typically, a given sentence is first parsed into a semantic representation and then re-produced in order to see whether the produced sentence is equal or sufficiently similar to the input sentence. A computational formalism is particularly useful if teams of linguists are working on a broad coverage grammar and have to share grammar fragments and build further on the work of others. And of course, a computational formalism is central to any efforts to build natural language processing applications that make use of ‘deeper’ and linguistically motivated models of language.
However the good news is that there are a number of active research programs trying to fill this gap. Some examples are: Dynamic Construction Grammar [Dominey and Boucher(2011)], Embodied Construction Grammar [Bergen and Chang(2003)], Fluid Construction Grammar [Steels(2011)], Sign-based Construction Grammar [Boas and Sag(2012)] and Template Construction Grammar [Barres and Lee(2014)]. The first symposium on computational construction grammar and frame semantics took place in the spring of 2017 (http://www.aaai.org/Library/Symposia/Spring/ss17-02.php) and we see a growing number of workshops and tutorials being planned.

The various computational platforms that have been proposed have obviously many things in common, such as the fundamental principle that signs, i.e. pairings of forms with meanings or functions, form the core unit of grammar. This contrasts with the notion, widespread in generative grammars, that the syntactic component must be self-contained and uniquely devoted to syntactic issues with semantics and pragmatics delegated to other components. Consequently, constructional platforms are transducers, mapping meanings to utterances and utterances to meanings, rather than generative devices that generate the set of all syntactically well-formed possible utterances that may occur in a language. But there are also significant differences, mostly due to the computational mechanisms with which each platform has been built. These range from logic-based, object-oriented, constraint-based and rule-based representations to neural network like structures. Each of these formalisms is still undergoing substantial evolution as their developers try to tackle new aspects of language or new application areas.

This paper focuses exclusively on Fluid Construction Grammar (FCG). This formalism started to be developed already around 2000 and has since seen major redesigns and re-implementations. It has been operational for many years and tested on a wide range of fragments for many different languages. A broad coverage grammar for English is under development. The FCG-system (Figure 1), which is capable to parse and produce utterances given a grammar, features also a powerful graphical interface and many other tools, including a server to run FCG remotely and components for quickly building web demonstrations. There is also a facility for running examples remotely on a cloud server, called FCG-interactive (see: https://www.fcg-net.org/fcg-interactive/). This is extremely helpful to test and explore FCG through the web without having to install any software.

The present paper introduces some of the basic ideas behind FCG as succinctly and didactically as possible and is intended as an introduction to the other papers in this volume which show how FCG can be used to formalize and discuss specific linguistic issues, such as long distance dependencies or fluid categories. The paper has an associated web demonstration (accessible through http://www.fcg-net.org/demos/basics-of-fcg) which implements all the constructions discussed here. Indices of symbol-names as used in the examples in this paper may differ from the web demonstration as they are
Figure 1: The FCG-engine reconstructs meanings from utterances or translates meanings to utterances using an inventory of construction schemas. The schemas are either defined by a linguist or grammar engineer or derived by a machine learning system from corpus data and other resources. The operation of the FCG-engine (also called the FCG-interpreter) and the outcome of processing can be inspected through the FCG-interface.

dynamically generated by the FCG-engine as you run the demonstration. Comparisons to other formalisms are found in [Chang and Micelli(2011)] and [van Trijp(2013)].

Due to the rich nature of the structures underlying human languages, a computational platform for construction grammar requires a lot of expressive power and must support complex structural operations. It should support interfaces to follow the details of language processing and provide tools for managing large inventories of constructions. Processing should be efficient enough to handle tens of thousands (if not hundreds of thousands) of constructions. It is therefore inherently difficult to summarise any constructional platform in a single paper. Here I just give a very brief summary of the core ideas behind FCG, abstracting as much as possible away from implementation details. If you are curious for more, I refer to the growing literature on FCG which exposes the underlying design principles, describes the basic mechanisms more fully, and provides many more concrete examples. See [Steels(2011)], [Steels(2012b)] and the other papers in this special issue. [Steels and Szathmáry(2016)] sketches a view on FCG as a biological system. Web resources accessible through http://www.fcg-net.org/ contains downloadable materials and various web demonstrations associated with papers that published grammar fragments, an on-line course, and more background papers.

The main contribution of the present paper is a new notation for FCG, compared to earlier publications. Since the first draft of this paper, written in 2013, this new notation has been implemented by Paul van Eecke and
has meanwhile become the standard notation used in FCG - even though the old notation is still used ‘under the hood’ as an internal representation. The new notation is used in all the papers in this special issue.

Here are two important preliminary points: (A) FCG is as neutral as possible with respect to the linguistic theory a grammar engineer may want to explore, as long as the analysis adopts a constructional perspective. The set of possible grammatical categories, types of units, relations between units, possible constructions, relations between constructions, etc. is entirely open. For example, one can explore a verb template approach just as easily as a constructionist approach to argument-structure [Goldberg(2015)], emphasize phrase structure versus dependency grammar or vice-versa, use predicate calculus or a frame-based formalism for semantics, etc. Therefore, when I show examples in this paper, they should be seen as no more than examples, more precisely didactic examples. They are not limited because I underestimate the competence of the reader, but because of space limitations. Moreover, the examples are not to be interpreted as claims that this is the only or the best way in which a particular linguistic phenomenon has to be handled.

There are two reasons for taking this stance:
(i) The history of formal and computational analyses shows that there is seldom agreement in the literature on how particular linguistic phenomena have to be dealt with. This might simply be because there are often many ways, just like the same computational function can often be achieved through many different algorithms and implemented in very different programming styles.
(ii) Many researchers are ultimately interested in how grammars are built and acquired autonomously by learning systems. Different learning algorithms will undoubtly come up with different grammars depending on what learning operators they use or what sequences of examples they get. So a formalism has to be as open as possible.

Of course, all this does not mean that one should refrain from performing detailed linguistic analysis and formalisation to see whether the representational and computational mechanisms of FCG are adequate, efficient, and ‘user-friendly’ enough to build large-scale natural grammars.

(B) The second preliminary point is that FCG does not make any claims about biological realism or cognitive relevance. A construction grammar expressed in FCG is viewed as a linguistic theory of the systematicity underlying a particular language. However this systematicity is usually only partial as a language is a complex adaptive system undergoing constant change and exhibiting wide variation in the population of its language users. It is debatable whether language users have a representation of the full systematicity of their language. They probably use a mixture with a very large store of ready-made form-meaning pairs with some general abstract patterns on the one hand and heuristic strategies for flexibly extending or adapting
these patterns on the other. Language users are certainly not able to articu-
late the linguistic system underlying their performance explicitly in a conscious manner. The main challenge addressed by FCG is therefore to allow construction grammars to be formally described and made the core component of language processing systems. It attempts to provide grammar engineers with adequate tools to scale up their grammars so that they have a broad coverage. Writing operational definitions of constructions is a very non-trivial affair, particular if the same definition has to be usable both in parsing and production, and if the grammar needs to be scaled beyond a dozen constructions.

The rest of the paper introduces first the semiotic cycle (section 2) and then the basic units of FCG: transient structures (section 3) and construction schemas (section 4). Section 5 adds meaning to this picture and section 6 introduces a slightly more complex example of the double object (or ditransitive) construction.

2 The Semiotic Cycle

2.1 Components of the semiotic cycle

A language interaction requires that speaker and hearer go through a set of processes, usually called the semiotic cycle [Steels(2015)], and computational models need analogues of these processes. To produce an utterance, the speaker needs to achieve the following tasks [Levelt(1989)]:

- **Perception and Action:** Both speaker and hearer must perceive the shared context to build and maintain a model of this environment and the actions that make sense in their interaction.

- **Conceptualization:** The speaker must decide which communicative goal should be achieved and has to conceptualize reality in order to come up with the meanings he could convey to achieve that goal. For example, the speaker might want to draw the attention of the hearer to an object in the shared context.

- **Production:** The speaker must translate meaning into an utterance based on the lexicon and grammar of the language, which implies building a set of intermediary semantic and syntactic structures. Input to production is the meaning to be expressed.

- **Rendering:** The speaker uses the result of the production-process to render the utterance through written or spoken forms.

To achieve understanding, the listener needs to accomplish the following tasks:
• **De-rendering:** The listener must extract basic observable features from the sounds emitted by the speaker or from the written text. Output is a description of the sequence of phonemes, intonation and stress structures, or the set of words ordered sequentially.

• **Parsing:** The listener must reconstruct the meaning suggested by the utterance using his own lexicon and grammar. This includes not only recovering the grammatical structures underlying the utterance, a process usually called *syntactic parsing*, but also recovering from these structures interpretable meanings, called *semantic parsing*. In construction grammar there is no strict distinction between these two processes.

• **Interpretation:** The listener must interpret this meaning in terms of his own world model and possibly act on it, for example, look at the object described by the speaker or point at it, or answer a question.

These various tasks are not performed in a sequential order. The speaker may already start rendering part of an utterance while still working out the grammar and the meaning to be expressed. He may have to focus his perceptual attention on some additional aspect of the scene to feed the conceptualization process. The listener is typically already interpreting meanings reconstructed by the comprehension processes and may start steering his perception to find which objects in the world are being talked about, while still having to process a large part of the utterance.

Control should be able to flow in a top-down and bottom-up manner. Speakers often self-monitor their own speech. They simulate how listeners would parse and interpret fragments of the utterance they are about to render in order to gauge the potential for communicative success, while listeners may simulate how the speaker conceptualizes and expresses meaning, in order to predict what the speaker is about to say or in order to learn new constructions.

It is also often the case that there is more than one hypothesis. There are usually multiple ways to achieve a communicative goal or to express a particular meaning and there are often multiple meanings and functions for the same words or grammatical patterns. So there is unavoidably a search space, i.e. a set of alternative solutions that needs to be explored. Hence the different processes in the semiotic cycle should be conceived as operating in parallel and are highly interlaced. FCG is primarily concerned with production and comprehension but can call upon the other processes in the semiotic cycle at any time.

Finally, we must envision that meta-level processes monitoring progress and trying to repair impasses or failures are running along with the basic processes of the semiotic cycle [Beuls, van Trijp and Wellens(2012)]. For example, when an unknown word is encountered, the parsing process should
be able to move to a meta-level, trying to gather enough information to continue processing and then extract information to make a good guess about a new lexical construction. These meta-level processes are not further discussed in this paper (see [?]).

2.2 Linguistic Pathways

It is common in computational linguistics to view production and parsing in terms of a chain of consecutive operations over a linguistic structure, called a transient structure in FCG parlance. A sequence of transient structures on a particular execution chain is called a linguistic pathway. The FCG-engine orchestrates the whole process using an inventory of construction schemas called an FCG-grammar. The FCG-grammar contains lexemes (construction schemas for lexical items), morphological construction schemas, idioms (construction schemas for idiomatic or often occurring expressions) and grammatical construction schemas, but there is no formal or computational difference between them. A graphical user-interface (the FCG-interface) is available for tracing operations and for inspecting transient structures and construction schemas (See examples at http://www.fcg-net.org/demos/basics-of-fcg.)

In parsing, the initial transient structure contains all information derived from the input utterance by derendering: which words, morphemes, ordering relations, intonation and stress patterns are present. Successive application of construction schemas then expands the transient structure to derive the meaning of the input utterance. It is possible to supply information about the utterance in a step-wise fashion from left to right and each time the FCG-engine does as much as it can with the available construction inventory and the input so far. It is also possible to provide all information at once.

In production, the initial transient structure contains the meaning to be expressed and then consecutive applications of construction schemas expand the transient structure until all information is available to render the utterance. Again, the target meaning could be supplied all at once, or it could be provided step-wise as the conceptualization process unfolds, partially based on demands from the production process.

Operations that expand transient structures are of three types:

1. **Application of a construction schema.** The FCG-engine has access to an inventory of construction schemas. Each schema operationally defines a particular construction by specifying a set of constraints on its construction elements. To make schema application efficient, some constraints are considered to form a lock that acts as a gate keeper, and the other constraints then form the contributor. This is inspired by the metaphor of a lock and a key, also used in biology to describe the activity of enzymes in molecular pathways [Steels and Szathmáry(2016)]. The transient structure is the key that can open the lock. If the key fits with the lock, the con-
struction schema opens and the transient structure is expanded with the constraints from the contributor (See Figure 2). A lock therefore represents the cues that trigger the activation of the construction schema.

We will see later that there are in fact two locks, a comprehension and a production lock, so that construction schemas can be applied in both directions: from form to meaning (for comprehension) and from meaning to form (for production). Moreover when the comprehension lock is used to open the construction schema the constraints in the production lock become part of the contributor and, vice-versa, when the production lock is used to open the construction schema, the constraints in the comprehension lock become part of the contributor.

Figure 2: An FCG construction schema consists of a set of construction elements and constraints on these elements in the form of features and values. The constraints are divided into a lock, acting as gate-keeper or cue, and a contributor. The transient structure is like a key. When it matches, it can open the lock, and then the constraints from the contributor are added to the transient structure.

Construction schemas are applied by the FCG-engine using an operator called Apply (defined in the appendix). Unification forms the core of this application process. Unification is one of the fundamental algorithms of computer science [Knight(1989)] and the basis of logic programming languages such as PROLOG. It compares two expressions, a pattern and a source, both possibly containing variables, and tests whether a suitable set of bindings of the variables can be found so that the pattern is a subset of the source. Apply decomposes into two functions: Match and Merge. Match compares two expressions - a transient structure and a lock and computes a set of bindings of the variables that would make the lock a subset of the transient structure. If there is a fit, then information from the contributing part of a construction schema, called the contributor, is added to the transient structure, through a merging process, called Merge, also operationalized using
unification.

2. **Expansion of the transient structure based on semantics.** In realistic language processing, it happens quite often that a fitting construction schema is not (yet) available in the listener’s construction inventory. In this case, the comprehension system can fall back on semantics, by calling the interpretation process which has access to the world through a perceptual system or to common sense knowledge and knowledge of the context, in order to suggest possible ways in which words or phrases could be linked together. This kind of expansion based on semantics make it often possible to continue a comprehension process even if construction schemas are missing or the input is ungrammatical.

3. **Activation of a meta-operator.** Meta-operators allow speakers and listeners to expand or stretch their grammars in order to deal with gaps or unexpected situations. The meta-operators consist of diagnostics which track whether production or comprehension is running smoothly and signal a problem when it appears, for example, when there is a word or grammatical construction schema missing to express a particular meaning, when all words have been parsed but the derived syntactic structure fragments cannot be fully integrated, whether there is combinatorial search in parsing, etc. Repairs become active based on the output from these diagnostics and they introduce new construction schemas, recategorize existing words, etc.

In this paper I focus exclusively on (1), i.e. the application of construction schemas, although the FCG-interpreter smoothly handles (2) and (3) as well. FCG has various utilities to write diagnostics and repairs, including an *anti-unification* operator that discovers commonalities between construction schemas and differences between construction schemas and a *pro-unification* operator that specializes a generalized construction schema to make it more specific again (see [?]).

3 **Transient Structures**

One of the key characteristics of construction grammar is its “insistence on simultaneously describing grammatical patterns and the semantic and pragmatic purposes to which they are dedicated” [Fillmore(1988)][p.36] so transient structures, as well as construction schemas, need to be able to represent information from a multitude of different perspectives. If we see a complete language system (relating sound to meaning) as a big cake, then traditionally, linguistics cuts the cake in different horizontal layers: phonology, morphology, syntax (with layers for phrase structure grammar, functional grammar, dependency grammar, case grammar), semantics (with layers for propositional structure, argument structure, temporal structure, modality) and pragmatics (including information structure and dialog management). Each layer is assumed to have its own autonomy, its own representational
structures, and its own set of rules with modular interfaces between the layers. The distinction between the c-structure (phrase structure) and the f-structure (functional structure) in Lexical Functional Grammar is a good example [Bresnan(2001)]. Instead, construction grammarians like to cut the cake in a vertical fashion, freely combining information across different layers. For example, the same construction schema may contain phonological constraints, syntactic constraints, aspects of meaning, as well as pragmatics. The layers should be seen as perspectives on the utterance.

3.1 Feature Structures

FCG represents transient structures using (extended) feature structures, familiar from many other contemporary grammar formalisms, particularly HPSG [Sag et al.(2003)]. An FCG feature structure consists of a set of units, that may correspond to lexical items (words, stems, affixes) or groupings of these items (for example phrases but other groupings are possible as well). Each unit has a name. The name has to be unique within the context of a particular transient structure, but the same name can be used in different transient structures.

Each unit has a feature-set which is a set of features and values. There is no ordering assumed among the features. The features provide information from different linguistic perspectives. Aspects of the meaning or of the form of an utterance are represented as features and sprinkled over different units in comprehension or production and collected at the end of processing to provide the total meaning or total form of the utterance.

Units. Feature structures are familiar from other grammar formalisms such as Unification Grammar [Kay(1984)] or HPSG [Sag et al.(2003)], but there are some small but important differences in their usage here. In classical feature structure formalisms [Shieber(1986)], there are no explicit units, only feature-sets, and consequently no names to address units directly, only path descriptions. As we will see, reifying units makes it possible to obtain clearer structure and names of units make it possible to directly refer to units, for example to represent what the subunits are of a unit, how units are ordered, or which dependency relations hold between them.

FCG is entirely open to what counts as a unit and what the possible features and values of units are. It depends on the linguistic theory the linguist wants to experiment with, or what categories and structures a learning algorithm might come up with. Moreover the units in a transient structure form a set and each unit is accessible independently within that set through its name, without traversing a path in a tree. This gives great efficiency and enormous flexibility to deal with languages that do not have a strict phrase structure or use strict sequential ordering, and it allows multiple structures to be imposed on the utterance (e.g. dependency structure and constituent structure).
Symbols. Feature structures use a lot of symbols: for unit-names, feature-names, values, predicates, objects in the domain of discourse, etc. In this text, they are written in courier font. A very small set of symbols are special and known to the FCG-interpreter, however most of them are specific to a specific FCG-grammar. The name of these symbols does not (indeed should not) play any role in its functioning. Their meaning is uniquely determined by its occurrence in all the feature structures and construction schemas in which they appear. For example, it is not by calling a unit subject or a feature number that the FCG-interpreter knows what these symbols stand for. A unit which is acting as the subject could just as well be called u-85773 and the gender feature could be called b444adb, as long as the same symbol-name is used everywhere else where reference is made to the same unit. But it is of course much better to use symbol-names that make sense for us, because that makes it easier to follow what a symbol is about. Once grammars become more serious, there will be tens of thousands of symbols, and even when processing a moderate-sized utterance, hundreds of novel symbols are created and used within a transient structure. Choosing good symbol-names is one of the first rules of good programming practice and it applies here as well.

Often we need many tokens of the same type, for example, there may be several adjectives in an utterance, in which case we use a type-name followed by an index like adjective-1, adjective-2, etc. The indices, as well as the indices of variables, categories, objects or other symbols of which there are many tokens, are automatically constructed by the FCG-engine.

Values. Values of features can be of different types: an atomic symbol, a set, a sequence, a feature-set, a Boolean expression, a predication (i.e. predicate-argument expression), or an expressions with special operators, to be defined soon. The type of value assumed for a particular feature is declared explicitly, and the default is an atomic value or feature-set. Only atomic symbols, sets, predications and feature-sets are used in this introductory paper. Sets are written with curly brackets { and }. Sequences are written with square brackets [ and ]. Their elements are written between commas. A predication has a predicate and a number of arguments as in: predicate(arg1, ... , argn).

Variables. Variables are written with a question-mark as the first character of the symbol name, so that we see immediately that the symbol is a variable. Examples are: ?NP-unit or ?number. Any element in a feature structure (symbols, units, sets, formulas, feature-sets) can be a variable, except feature-names. It is recommended that variables have understandable rather than arbitrary names, just for us to make sense of the role of the variable. FCG variables have the same behavior as logic variables in logic programming languages.

We follow tradition by representing a list of variable bindings, also called a substitution, as a set of dotted pairs:
Each unit in a transient structure is represented as a box with the name of the unit on top and the feature-set of the unit below it. The semantically-oriented features, called the \textit{semantic pole}, are usually written above the syntactically-oriented features, called the \textit{syntactic pole}, but for the FCG-engine there is no fixed ordering of the features and no \textit{a priori} defined set of features that must occur obligatory. The FCG user interface can display the syntactic and semantic poles of units separately, which is helpful when inspecting complex transient structures, but in this paper they are always displayed as one set.

Ex. 1 is a possible transient structure for the utterance “he bakes”:

\begin{tabular}{|c|}
\hline
\textbf{he-unit-2} \\
\hline
\end{tabular}

\begin{tabular}{|c|}
\hline
\textbf{bakes-unit-2} \\
\hline
\end{tabular}

There are two units, one for each word. The units are called \textit{he-unit-2} and \textit{bakes-unit-2}. (Recall that the indices are made up by the FCG-engine. Unit-names could just as well have been \textit{he-unit-5} and \textit{bakes-unit-123} for example.) We see here semantics-oriented features, such as \textit{meaning} with a \textit{referent}, \textit{args}, and \textit{predicates} and \textit{sem} (semantic properties) with \textit{sem-cat} (semantic categories), \textit{sem-fun} (semantic function) and \textit{frame} (semantic valence frame). We also see form and syntactic-oriented features, such as \textit{form} which specifies the \textit{string}, and \textit{syn} (syntactic properties) with \textit{syn-cat} (syntactic categories), \textit{syn-valence} (syntactic valence) and \textit{syn-fun} (syntactic function). \textit{he-unit-2} similarly represents various aspects of an occurrence of the word “he”, including information about meaning, semantic features, form, and syntactic features. Note that this is just one way to capture all this information. A grammar engineer is entirely free
about which features to use or how to structure them.

Units typically have a form-feature which collects information about the form in terms of a set of predicates. One of these predicates is `string` (in the case of the comprehension or formulation of written language). It takes two arguments: the unit and the string itself, as in

\[
\text{form: \{string(he-unit-2,\text{"he"})\}}
\]

where `he-unit-2` is the name of a unit.

The same form-feature also stores information about sequential ordering using two predicates: `precedes` and `meets`. These predicates have three arguments, two units which are left and right from each other and a unit which acts as a context. A unit \( u_1 \) precedes a unit \( u_2 \) iff the wordform(s) of \( u_1 \) precede those of \( u_2 \). A unit \( u_1 \) meets a unit \( u_2 \) iff the wordform(s) of \( u_1 \) immediately precede those of unit \( u_2 \). For example, to represent that the form of `he-unit-2` (namely “he”) immediately precedes the form of `bakes-unit-2` (namely “bakes”) we write:

\[
\text{form: \{string(he-unit-2,\text{"he"}), string(bakes-unit-2,\text{"bakes"}), meets(he-unit-2,bakes-unit-2,np-unit-5)\}}
\]

\( np-unit-5 \) is the context in which the meets-relation holds. In the stream-lined notation used in this paper, the ordering relations are not represented explicitly but a < sign is put between the relevant units for the precedes-relation and a ≤-sign for the meets-relation.

The units in a transient structure form an unordered set. Various relations can be imposed on the units and they are explicitly represented using features and values. For example, hierarchical phrase structure can be represented with a feature `constituents` which contains as its values the list of phrasal constituents of a unit, or if you want to represent dependency relations, you can do it with a feature `dependents`. All this is up to the grammar designer. FCG does not insist on trees (and certainly not on binary trees). Some perspectives (such as phrase structure) impose a tree on units but other perspectives (such as dependency structure) impose a graph. Sometimes (particularly in formulating) it is not yet known how units relate to each other, so it would not be possible to draw a tree or a graph.

Dozens more features are typically required to represent all information needed in comprehension and formulation. The different features have been structured here into different subsets and features like `meaning`, `sem`, `syn`, `form`, etc., that have as value another set of features. However, in the examples later in the paper, I will use a single feature-set to make the examples fit on a page. For example, `he-unit-2` (Ex.1) is represented as:
What features are used, how they are combined into feature-subsets and how they are labeled is entirely in the hands of the grammar designer, although obviously teams of grammar designers should try to reach a consensus to make exchange and combination of grammar fragments possible.

### 4 Construction schemas

A construction schema can be used to expand any aspect of a transient structure from any perspective and it can consult any aspect of this transient structure to decide how to do so. Construction schemas are not merely intended to be descriptive, they should provide enough information to operationalize a construction. A construction schema is adequate if it triggers only in appropriate circumstances and expands the transient structure only in the right way. It turns out that it is very non-trivial to come up with operational construction schemas, particularly because they usually have side effects on the triggering of other construction schemas. So it is not possible to design construction schemas in isolation.

FCG construction schemas use the same representation as transient structures. They are formulated in terms of named units which represent the constructional elements. The features and values associated with each unit represent constraints. Any symbol in a construction schema may be variable, including the name of a unit, except feature names, like referent or syn-fun. In fact the unit names will all be variables because they have to bind to concrete units in transient structures.

Although construction schemas are represented using feature structures, they are (more) abstract than transient structures because many of its elements are variables instead of concrete elements. Some of the features are not specified or only partially specified, and a construction schema typically contains fewer units. The inventory of construction schemas available to a language user is called a constructicon. As a first approximation, we assume that all construction schemas form an unordered set (but see section 7).

To understand better how the FCG-engine uses construction schemas, I
will start from rewrite rules, a formalism that every computational linguist is familiar with, and progressively introduce innovations to arrive at FCG construction schemas. I will also use first a simple example from phrase structure grammar, again, because phrase structure is familiar to all readers. But keep in mind that functional grammar, dependency grammar, case grammar, information structure or any other grammatical perspective can be accommodated and used just as easily, and that construction schemas can combine information from any of these perspectives.

4.1 Rewrite Rules using Feature Structures

A typical set of rules in a very simple phrase structure grammar for English might be:

\[ \begin{align*}
  &1 \text{ NP} \rightarrow \text{art noun} \\
  &2 \text{ art} \rightarrow \text{the} \\
  &3 \text{ noun} \rightarrow \text{girl}
\end{align*} \]

Such rules are usually interpreted in a generative way, namely, if you have the symbol(s) on the left-hand side of a rule then you can replace them by the symbols on the right-hand side. For example, if you start from NP as the initial symbol, you can derive the sentence “the girl” by application of [1] then [2] and then [3]:

\[ \text{NP} \xrightarrow{1} \text{art noun} \xrightarrow{2} \text{"the" noun} \xrightarrow{3} \text{"the girl"} \]

FCG is not intended for generating random sentences but for modeling comprehension and formulation processes. Even though the term generation is often used for formulation as well, we make a strict distinction here. Formulation is coming up with an utterance (and its underlying structure) that expresses a particular meaning according to the constraints imposed by the grammar. Generation is coming up with a random example utterance (and its structure) that conforms with the constraints defined by the grammar.

A rewrite grammar can be used in the syntactic parsing phase of comprehension by applying rules in the reverse direction: An occurrence of the symbol(s) on the right-hand side of a rule is replaced by the symbol on the left-hand side. For parsing “the girl”, we get the following chain of rule applications:

\[ \text{“the girl”} \xrightarrow{3} \text{art “girl”} \xrightarrow{2} \text{art noun} \xrightarrow{1} \text{NP} \] (Ex.3)

However a rewrite grammar is too underconstrained to be used in formulation, as soon as the same symbol occurs more than once on the right-hand side of a rule. For example, NP can be rewritten in hundreds of different ways (NP \( \rightarrow \text{N}, \) NP \( \rightarrow \text{Art N}, \) NP \( \rightarrow \text{NP prep NP}, \) etc.). A generator can just pick randomly one of these, but a formulator has to choose one option, based on semantics and many other criteria that are not expressed in a rewrite grammar.
In the late nineteen seventies, almost all computational formalisms started to replace atomic symbols by feature structures, as pioneered in logic-based (definite clause) grammars [Pereira and Warren (1980)] and unification-based grammars [Kay (1984)]. FCG does this too. For each atomic symbol there will be a unit. The feature-set associated with this unit specifies its properties. To represent phrase structure, the relevant properties are the string and lexical category for word units, and the phrasal category, the subunit-relations and their ordering for phrasal units. The application of a rule no longer replaces the occurrence of symbol(s) on its right-hand side by the symbol on the left-hand side, but rather adds the information on the left-hand side to the transient structure.

Starting from two units `the-unit-1` and `girl-unit-1` representing “the” and “girl” respectively, the same parsing sequence as in Ex.1 then looks like this:

<table>
<thead>
<tr>
<th>```</th>
<th>```</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>the-unit-1</code></td>
<td><code>girl-unit-1</code></td>
</tr>
<tr>
<td>form: `{string(</td>
<td>form: `{string(</td>
</tr>
<tr>
<td><code>the-unit-1&quot;,&quot;the\&quot;))</code>,</td>
<td><code>girl-unit-1&quot;,&quot;girl\&quot;))</code>,</td>
</tr>
<tr>
<td>lex-cat: art</td>
<td>lex-cat: noun</td>
</tr>
</tbody>
</table>

NP-1
---
phrasal-cat: NP
 constituents: `{the-unit-1, girl-unit-1}`

Keep in mind that the units in a transient structure form a set and that sequential orderings are written using the \(\leq\)-sign.

The use of feature structures as a way to represent the current state of parsing has a number of significant advantages. A parser can dispense with an external memory device like a stack because all information to continue the computation is contained in the transient structure itself. It is also easy to set up a search space for comprehension or formulation. Each node in the search space simply consists of a transient structure and there is an outgoing branch for each construction schema that matches with this transient structure. If more than one construction schema can expand a transient structure, then several nodes are added to the search space and different
strategies (depth-first, best-first, etc.) can be applied. The FCG-engine is able to handle search spaces and it is straightforward to implement different search strategies. By default, the FCG-engine uses a best-first heuristic search with backtracking, meaning that the potential success of each branch is computed based on criteria such as the score of the construction schema (which is based on past usage), the scope (schema’s that can handle more of the input are preferred), interpretability given the current situation model (in comprehension), etc., and then the best hypothesis is explored first. When this hypothesis leads to a dead-end, the best alternative hypothesis is pursued.

4.2 Representing construction schemas

The next step is to translate rewrite rules into feature structures as well. I start calling rules from now on construction schemas and talk about construction application instead of rule application. I will also change the direction of the arrow so that it is clear that we are no longer talking about standard generative rewrite rules. The right-hand side of the construction schema is called the lock. The transient structure has to fit with the lock as established by the matching process (Match). The left-hand side is called the contributor and it contains information that will be added to the transient structure by the merge operation (Merge), once the lock has been opened.

The first step, to translate the rules themselves, is straightforward. The atomic symbols in a rule are replaced by units with names, features and values. Instead of names of specific units (as in a transient structure), the unit-names are variables that are to be bound to the concrete units found in a transient structure, because the construction schema is an abstraction that should be applicable to many different transient structures.

The second step is to add more sophistication to the application process. Instead of simply comparing two symbols, the FCG-interpreter has to find for every unit on the right-hand side of a construction schema a matching unit in the transient structure, and, if this is the case, the information on the left-hand side is merged with the information already present in the transient structure. As mentioned earlier, information is added. The matching units are not replaced, as in parsing with traditional rewrite rules. Moreover if the merging leads to a conflict, the application process still fails.

The details of the matching and merging processes are tricky, partly because of the logic variables. However they are entirely similar to the kind of unification found in logic programing languages, familiar to many computational linguists. Details are presented in the appendix.

Con.1a is a translation of rule (1) in the rewrite grammar given earlier. The unit-names are ?NP-unit, ?art-unit, and ?noun-unit. Although units in construction schemas have the same characteristics as in transient structures, I will use left and right square brackets (as in Con.1a) instead
of boxes (as in Ex.4) for the different units, to make the distinction visually clear.

\[
\begin{array}{c}
\text{\?NP-unit} \\
\text{\text{\text{phrasal-cat: NP}}} \\
\text{\text{\text{constituents: \{\?art-unit, \?noun-unit\}}} \\
\text{\text{\text{number: \?number}}}
\end{array}
\] ←

\[
\begin{array}{c}
\text{\?art-unit} \\
\text{\text{\text{lex-cat: article}}} \\
\text{\text{\text{number: \?number}}}
\end{array}
\] ≤

\[
\begin{array}{c}
\text{\?noun-unit} \\
\text{\text{\text{lex-cat: noun}}} \\
\text{\text{\text{number: \?number}}}
\end{array}
\] (Con.1a)

The right-hand side defines two units, \?art-unit and \?noun-unit, which have the values article and noun for their lex-cat feature. The left-hand side introduces a new unit for a noun-phrase, which has a feature constituents filled by a set with two units: \?art-unit and \?noun-unit. The \?NP-unit has the value NP for its phrasal-cat feature.

Feature structures have become so widespread because they can easily handle all sorts of phenomena which defy traditional rewrite rules, for example, representing grammatical features (such as gender, number, case, tense, aspect, etc.) and dealing with grammatical agreement and percolation of feature values from constituents to their parent phrasal units.

For example, English requires agreement for number and person between subject and verb and then percolation of number, person and gender from the head of a noun-phrase to the noun-phrase as a whole. We can express this easily by using the same variables in each unit participating in agreement or percolation. This is illustrated in Con.1a. The \?NP-unit, the \?art-unit and the \?noun-unit all have the same number-value because they share the variable ?number. This constrains the application of this construction schema. It now can become active only when the article and the noun bind to the same number-value ?number and this value then percolates as the number-value of the \?NP-unit in the merge-phase.

Now we make another seemingly trivial but very important step: We allow that a unit appearing on the right-hand side re-appears on the left-hand side. In this case, no new unit needs to be made but the information on the left-hand side will be added to the already existing unit when the construction schema is applied.

This is illustrated in Con.1b. The \?art-unit and \?noun-unit not only appear on the right-hand side but also on the left-hand side with additional information about the function and dependencies of these units, namely that they are the determiner of the noun-unit and the head of the noun-phrase respectively. This construction schema builds both the phrase structure and the functional structure (using the categories determiner and head).
The fact that units on the left-hand side can add information to existing units has an important consequence. The existing unit in the transient structure might be incompatible with what the construction schema wants to add. Earlier on we have seen that the matching operation, which compares the right-hand side of a construction schema with the transient structure, could either block or allow a construction schema to be considered further. Now the merging operation, which adds information from the left-hand side to the transient structure, can also block further application of the construction schema as well. For example, if for some reason the ?art-unit had already another filler for syn-fun then the application of the construction schema would fail.

By allowing the same units to appear on both sides of the arrow, we break away from a possible generative usage of the grammar. Generation requires that the symbols on the left-hand side are either the initial symbol or symbols that have been added earlier. This is no longer the case here. So \( X \leftarrow Y \) can no longer be interpreted as “\( X \) can be rewritten as \( Y \)” but rather as “\( X \) can be projected (added/imposed) on the transient structure which is subsumed by \( Y \)”.

There is a further important step which distinguishes FCG construction application from standard rewrite systems. Even units that did not occur on the right-hand side (in this case the \( ?\text{NP-unit} \)) might already exist in the transient structure, in which case they are not created anew but the information supplied by the construction schema is simply added. For example, it is possible that a construction schema makes a prediction that there is verb phrase coming before there is any evidence in the input, and then when this evidence arrives (for example a verb appears in the input) the predicted verb phrase gets further filled in. This feature of FCG is computationally possible because the merging process carries out first a matching process to find whether any unbound but matching units on the left-hand side already appear in the transient structure, before adding new features and values.
4.3 Interfacing

The de-rendering task in the semiotic cycle transforms the input into a list of elements which are sequentially ordered. These elements are usually words or possibly smaller units like morphemes or even phonemes. There could also be gestures or supra-segmental features that involve several words such as intonation contours. All this information is put in a unit called the root. This root-unit is a unit in every transient structure and it is unique, addressed with the symbol-name root.

De-rendering creates a unit-name for each of the elements in the input, but no real units yet. The root-unit has a form-feature that contains a description in terms of predicates of the form of the utterance. Which predicates are used is entirely up to the grammar designer. Typical example predicates are string, meets, precedes, etc. For the phrase “the girl”, the root unit looks as follows:

\[
\begin{align*}
\text{form:} & \{ \text{string}(\text{the-unit-1, "the"}), \\
& \text{string}(\text{girl-unit-1, "girl"}), \\
& \text{meets}(\text{the-unit-1, girl-unit-1, ?scope}) \}
\end{align*}
\]

Note that the scope of the meets-predicate is a variable, i.e. undefined yet. The scope will become bound to a concrete phrasal unit once that is known.

Construction schemas occasionally need to check whether features are present in the input. For example, lexical construction schemas typically look out whether a particular string is present in the root. These features are marked with the special operator \# (pronounced hash), which forces the FCG-engine to look in the root-unit. If matching predicates are found and they involve units which do not exist yet, those units are created, added to the transient structure, and given the matching predicates which are subsequently deleted from the root to avoid further triggering.

This sounds more complicated than it really is. An example clarifies. Here is the lexical construction schema for the word “the”, which looks for a string in an input utterance, such as “the girl”, described in the root-unit in Ex.5.

\[
\begin{align*}
\text{?the-unit} & \quad \text{referent: } ?\text{obj} \\
& \quad \text{lex-cat: article} \\
& \quad \text{number: } ?\text{number}
\end{align*}
\]

\[
\text{←} \quad \begin{align*}
\text{?the-unit} & \quad \text{# form: } \{ \text{string}(\text{?the-unit, "the"}) \}
\end{align*}
\]

When the FCG-engine is trying to find a unit in the transient structure that matches with ?the-unit as defined on the right-hand side of Con.2a, it will not find it. However it will find a matching predication in the root-unit in Ex.5 for the form-feature:

\[
\text{# form: } \{ \text{string}(\text{?the-unit, "the"}) \}
\]

?the-unit then gets bound to the-unit-1. Because this unit does not exist
yet in the transient structure, it is created and the matching predicates are added as in Ex. 6.

\[
\begin{align*}
\text{the-unit-1} & \\
\text{form:}\{\text{string(\text{the-unit-1}, \text{"the"})}\} & \\
\end{align*}
\]  

(Ex.6)

The information on the left-hand side of Con.2a is then moved from the root-unit to this new unit, yielding Ex.7:

\[
\begin{align*}
\text{the-unit-1} & \\
\text{form:}\{\text{string(\text{the-unit-1}, \text{"the"})}\} & \\
\text{referent: } ?\text{obj-1} & \\
\text{lex-cat: } \text{article} & \\
\text{number: } ?\text{number-1} & \\
\end{align*}
\]  

(Ex.7)

Note that the FCG-engine has renamed the variable names in the construction schema (?\text{number} and ?\text{obj}) to ?\text{number-1} and ?\text{obj-1} respectively (we stress again that these names are entirely arbitrary). This is necessary to keep variables apart when there are multiple applications of the same construction schema, for example if the same word appears more than once in a single utterance.

Here is another lexical construction schema, for the word “girl”:

\[
\begin{align*}
\text{?girl-unit} & \\
\text{referent: } ?\text{y} & \\
\text{sem-cat: } \{\text{animate, feminine}\} & \\
\text{lex-cat: } \text{noun} & \\
\text{number: } \text{singular} & \\
\end{align*} \leftarrow \begin{align*}
\text{?girl-unit} & \\
\# \text{form:}\{\text{string(?girl-unit, \text{"girl"})}\} & \\
\end{align*} \]  

(Con.3a)

Using the lexical construction schemas Con.2a and Con.3a as well as the noun-phrase construction schema Con.1b, the complete chain shown in Ex.4 can be achieved (as illustrated in more detail later).

5 Constructional Processing

One of the main features of construction grammar is that the basic unit of grammar, the construction, deals not only with syntactic issues but also with semantics, pragmatics and meaning. After all, a construction is a pairing of form and meaning/function. To incorporate this principle, we first need a way to represent meaning. Then we can address the question how meaning gets introduced in the comprehension process and how formulation can convert meaning into its grammatical expression.

5.1 Adding Meaning

FCG can be used with any kind of representation of meaning. In the work of our group on grounding language in the sensori-motor states of physical robots, we usually employ procedural semantics [Spranger et al.(2012)]. But
in what follows, I will use a predicate calculus style notation for meaning because that is undoubtedly already familiar to the reader. More precisely, I will use a very minimal lightweight form of the predicate calculus, using only predicates with arguments that are either constants or variables, and conjunctive combinations. A meaning representation then consists of a referent and a set of predications that implicate the referent as one of the arguments. Identifying the referent is why this meaning is expressed. The notion of referent is similar to that of the lambda-variable in Montague semantics and similar logic formalisms. All predicates are typed and we write: type(predicate, arg). This notation is illustrated in the following example:

referent: obj-2
predicates: \{state(sleepy, obj-2), person(girl, obj-2)\}

It describes a concrete object, obj-2, for which it is true that the predicate sleepy with type state is valid and the predicate girl with type person.

The meanings in the lexicon and grammar typically have variables because we do not yet know what the concrete objects are - indeed it is the task of the FCG-interpreter to find bindings between these variables and entities in the world model. Here is an example, with ?obj being a variable:

referent: ?obj
predicates: \{state(sleepy, ?obj), person(girl, ?obj)\}

N-ary predicates are decomposed into subpredicates. For example, the meaning of the word “bakes”, which might also be represented with a single predicate, as in bake(obj-6, obj-17, obj-15), is represented here with three predicates: bake, baker, and baked. The type of bake is action.

referent: obj-6
predicates: \{action(bake, obj-6), baker(obj-6, obj-17), baked(obj-6, obj-16)\}

Such a decomposition is quite common in computational linguistics and useful because it makes it clear what role the arguments play. It also becomes easier to add other predicates about the same event, such as time or location. Moreover it is a very common way to represent meaning in terms of frame-semantics.

In the above example, the referent of “bakes” is the bake-event itself. But it is possible to have other referents, for example, the baker (i.e. obj-17), as in “the baker of the cake”, in which case the meaning is expressed as:

referent: obj-17
predicates: \{action(bake, obj-6), baker(obj-6, obj-17), baked(obj-6, obj-16)\}

In formulation, the input consists of meaning, rather than form. But we use the same idea of a root-unit. The meaning that needs to be expressed is
added to this unit. For example, to start up the formulation of the sentence “he bakes a cake”, the root-unit looks as follows:

\[
\text{root} \\
\begin{align*}
\text{predicates:} \\
\{ & \text{person(male,obj-17)}, \\
& \text{action(bake,obj-6),} \\
& \text{baker(obj-6,obj-17),} \\
& \text{baked(obj-6,obj-16)} \} \\
\end{align*}
\]

(Ex.8)

5.2 The Formulation and Comprehension Lock

It is highly desirable that the same construction schema, without change, can be used for comprehension AND formulation. But formulation requires that construction schemas trigger mainly on the basis of semantic criteria, whereas comprehension requires that they trigger mainly based on syntactic criteria. This leads naturally to the idea that the constraints on each unit should be split into two: a formulation constraint and a comprehension constraint. The formulation-lock then consists of all the formulation constraints and the comprehension-lock of all the comprehension constraints.

- The formulation-lock is the gate-keeper in formulation. When the transient structure fits with the formulation-lock, all information from the comprehension lock is added as well as the information in the contributor (the left-hand side of the construction schema). So the comprehension-lock is not the gate-keeper in deciding whether the construction schema should be used in formulation, but its information will become part of the new transient structure and if it is incompatible with the transient structure it could still block the application of the construction schema.

- The comprehension-lock is the gate-keeper in comprehension. When the transient structure fits with the comprehension-lock, information from the formulation-lock as well as from the contributor are merged with the transient structure as long as this information is compatible with the transient structure.

The formulation-constraint for each conditional unit is written above the comprehension-constraint. When there is no constraint for a unit (either for formulation or comprehension), the symbol for an empty set, i.e. $\emptyset$, is used.

So we get the following construction schema template:

\[
\begin{array}{ccc}
\text{Contributing Unit}_1 & \ldots & \text{Contributing Unit}_n \\
\text{Constraints for Unit}_1 & \ldots & \text{Constraints for Unit}_n
\end{array}
\]
Conditional Unit_1

formulation-constraints_1

comprehension-constraints_1

... Conditional Unit_m

formulation-constraints_m

comprehension-constraints_m

Note that the same unit can appear both as a conditional unit and a contributing unit, which share the same name.

Here is an example of the lexical construction schema for “the”, which now has not only comprehension but also formulation constraints for the-unit on the right-hand side:

\[
\begin{align*}
\text{?the-unit} & \quad \text{?the-unit} \\
\text{referent: ?obj} & \quad \text{# meaning: \{definite(?obj)\}} \\
\text{lex-cat: article} & \quad \text{# form: \{string(?the-unit,"the")\}} \\
\text{number: ?number} & \\
\end{align*}
\]

Here is the lexical construction schema, for the word “girl”:

\[
\begin{align*}
\text{?girl-unit} & \quad \text{?girl-unit} \\
\text{referent: ?obj} & \quad \text{# predicates: \{person(girl,?obj)\}} \\
\text{sem-cat: \{animate, feminine\}} & \quad \text{# form: \{string(?girl-unit,"girl")\}} \\
\text{lex-cat: noun} & \\
\text{number: singular} & \\
\end{align*}
\]

And here is an example of a noun-phrase construction schema. The conditional meaning is rather limited here. Basically it establishes that the referent of article and the noun are the same by using the same variable-name for their referents (namely ?obj). And this variable is also used for the referent of the ?NP-unit.

\[
\begin{align*}
\text{?NP-unit} & \quad \text{?art-Unit} \quad \text{?noun-unit} \\
\text{referent: ?obj} & \quad \text{syn-fun:} \\
\text{phrasal-cat: NP} & \text{determiner: ?noun-unit} \\
\text{constituents: \{?art-unit, ?noun-unit\}} & \quad \text{head: ?NP-unit} \\
\text{agreement: ?number} & \\
\end{align*}
\]

\[
\begin{align*}
\text{?art-unit} & \quad \text{?noun-unit} \\
\text{referent: ?obj} & \quad \text{referent: ?obj} \\
\text{lex-cat: article} & \quad \text{lex-cat: noun} \\
\text{number: ?number} & \quad \text{number: ?number} \\
\end{align*}
\]

5.3 An example of comprehension

Suppose that we now try to parse “the girl”, starting from the input in Ex.5. The example can be inspected in Part I of http://www.fcg-net.org/demos/basics-
of-fcg. The application of the lexical construction schemas 2b and 3b yields the following:

\[ 2b,3b \rightarrow \]

<table>
<thead>
<tr>
<th>the-unit-1</th>
<th>girl-unit-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>predicates: {specificity(definite,?obj-1)}</td>
<td>predicates: {person(girl,?obj-2)}</td>
</tr>
<tr>
<td>form: {string(the-unit-1,&quot;the&quot;)}</td>
<td>form: {string(girl-unit-1,&quot;girl&quot;)}</td>
</tr>
<tr>
<td>number: ?number-1</td>
<td>number: singular</td>
</tr>
<tr>
<td>lex-cat: article</td>
<td>lex-cat: noun</td>
</tr>
</tbody>
</table>

(Ex.9)

Two new units have been built and both the formulation lock and the contributor (left-hand side) of the construction schema have supplied information for them. Now the noun-phrase construction schema (Con.1c) applies because its comprehension-lock matches, assuming the following binding-set:

\[ \{(?art-unit . the-unit-1)(?noun-unit . girl-unit-1) \ |
\quad (?obj . ?obj-1) (?obj . ?obj-2) \} \]

After merging we get the following transient structure:

<table>
<thead>
<tr>
<th>NP-unit-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>referent: ?obj-3</td>
</tr>
<tr>
<td>phrasal-cat: NP</td>
</tr>
<tr>
<td>constituents:</td>
</tr>
<tr>
<td>{art-unit-1, noun-unit-1}</td>
</tr>
<tr>
<td>number: singular</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>art-unit-1</th>
<th>noun-unit-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>referent: ?obj-3</td>
<td></td>
</tr>
<tr>
<td>predicates: {specificity(definite,?z-1)}</td>
<td>predicates: {person(girl,?z-1)}</td>
</tr>
<tr>
<td>form: {string(art-unit-1,&quot;the&quot;)}</td>
<td>form: {string(noun-unit-1,&quot;girl&quot;)}</td>
</tr>
<tr>
<td>syn-fun:</td>
<td></td>
</tr>
<tr>
<td>determiner: noun-unit-1</td>
<td></td>
</tr>
<tr>
<td>lex-cat: article</td>
<td></td>
</tr>
<tr>
<td>number: singular</td>
<td></td>
</tr>
<tr>
<td>syn-fun:</td>
<td></td>
</tr>
<tr>
<td>head: NP-unit-2</td>
<td></td>
</tr>
<tr>
<td>lex-cat: noun</td>
<td></td>
</tr>
<tr>
<td>number: singular</td>
<td></td>
</tr>
</tbody>
</table>

(Ex.10)

Note that the referent of the noun-phrase has been made equal to the arg-values of the constituents: ?obj was bound to ?obj-1 and to ?obj-2 and hence all occurrences of ?obj, as well as ?obj-1 and ?obj-2, have been replaced by a single new variable ?obj-3. Note also how the number-value singular coming from “girl” has propagated to the article and NP-unit. The number-value could have come also from the article (as would be the case in “a sheep”) or from the NP, due to agreement with the verb (as in “the sheep sleeps”). Variable-bindings flow in all directions and that makes them so powerful.
5.4 An example of formulation

Exactly the same construction schemas can now be used to go from meaning to form, except that the formulation-locks are now the gate-keepers. The starting point is the following root-unit:

```
root
| referent: obj-1 |
| predicates: {specificity(definite,obj-1), person(girl,obj-1)} |
```

(Ex.11)

The first step is the application of the lexical construction schemas. A lexical construction schema applies in formulation if its formulation-lock matches with predicates in the root-unit. If this is the case, then a new unit is created for the corresponding word. Information from the comprehension-lock on the right-hand side of the construction is merged into this unit and all other information from the contributor (on the left-hand side) is added as well. So we get the following two new units, with so far no ordering relations between them:

```
the-unit-2
| referent: obj-1 |
| predicates: {specificity(definite,obj-1)} |
| form: {string(the-unit-2,“the”)} |
| lex-cat: article |
| number: ?number-2 |

girl-unit-2
| referent: obj-1 |
| predicates: {person(girl,obj-1)} |
| form: {string(girl-unit-2,“girl”)} |
| lex-cat: noun |
| number: singular |
```

(Ex.12)

Now the noun-phrase construction schema Con.1c can apply. The binding set is

```
{(?art-unit . the-unit-2) (?noun-unit . girl-unit-2)
 (?number . singular) (?number . ?number-2) (?obj . obj-1)}
```

and it produces a similar structure as we saw in Ex.10:

```
NP-unit-3
| referent: obj-1 |
| phrasal-cat: NP |
| constituents: {the-unit-2, girl-unit-2} |
| number: singular |

the-unit-2
| referent: obj-1 |
| form: {string(the-unit-2,“the”)} |
| syn-fun: |
|    determiner: girl-unit-2 |
| lex-cat: article |
| number: singular |

girl-unit-2
| referent: obj-1 |
| form: {string(girl-unit-2,“girl”)} |
| syn-fun: |
|    head: NP-unit-3 |
| lex-cat: noun |
| number: singular |
```

(Ex.13)

Note how ?number-2 has been replaced by singular in the-unit-2 and this value percolated up to NP-unit-3. The referent of NP-unit-3 has become obj-1.
This particular example is also shown through the web demonstration as Example I. (see http://www.fcg-net.org/demos/basics-of-fcg). Other examples shown there involve intransitive (Example II) and transitive (example III) clauses.

6 Argument structure constructions

We now look at another example to further illustrate the basic mechanisms of FCG. The example concerns argument structure constructions, a topic that is closer to the interests of many construction grammarians than phrase structure. The web demo associated with this paper (see http://www.fcg-net.org/demos/basics-of-fcg) shows examples for intransitive and transitive argument structure constructions in Part II and Part III respectively. Here we discuss only Part IV, the double-object construction. The formalization is based on an earlier more detailed proposal, worked out by Remi van Trijp [van Trijp(2010)].

6.1 The double object construction

The double-object construction has two noun-phrases after the verb, as in: “He gave her the book.” It imposes the meaning of ‘causing to receive’, even if the verb itself does not express this already as part of its lexical meaning, which is the case for example in “He baked her a cake.” [Goldberg(1995)].

I ignore many subtleties of this construction, for example that the indirect object is usually animate or that the first noun phrase is necessarily shorter (a phonological constraint) or more topical (a pragmatic constraint) than the second. I will also ignore tense, aspect and agreement phenomena (although they are all worked out in van Trijp’s implementation) and morphology. Instead I focus only on the verb and how the construction can impose meaning beyond the meaning provided by lexical items.

Lexical constructions for “he”, “her”, and “(a) cake” are similar to what I discussed earlier. Here is the lexical construction for “bakes”:

\[
\begin{array}{l}
\text{\# predicates:} \\
\text{\{} \text{action(bake,?event),} \\
\text{baker(?event ?baker),} \\
\text{baked(?event ?baked)} \text{\}} \\
\text{\# form:} \\
\text{\{} \text{string(?bakes-unit “bakes”)} \text{\}} \\
\end{array}
\]

As expected, the lock, on the right-hand side, contains a verb-unit whose formulation-constraint requires the predicates describing a bake event:

\[
\begin{array}{l}
\text{\# predicates:} \\
\text{\{} \text{action(bake,?event),} \\
\text{baker(?event ?baker),} \\
\text{baked(?event ?baked)} \text{\}} \\
\end{array}
\]
and whose comprehension-constraint requires the string “bakes” (in fact it should be the stem “bake-” but I gloss over morphological details here).

The contributor, on the left-hand side, imposes a semantic valence structure (a case frame) for the same verb-unit with an actor and an undergoer. The actor is ?baker and the undergoer is ?baked. These variables are the same as those in the lock, so that if they get bound, they are also bound in the case frame as well. The contributor also introduces a syntactic valence for the verb-unit with roles for a subject and an object. The subject or direct-object cannot be known based on this construction alone, and so variables, ?subj and ?obj, are introduced whose values will have to be bound by other constructions.

Next, Con.5 is the definition of the double object construction itself. It has to handle phrases like “she gave him a book”, where the receiver is already included in the syntactic and semantic valence of the verb but also “she baked him a cake” where the receiver has to be added by the construction. The presence of a receiver in the semantic valence frame of the verb is therefore not obligatory, but it is added by the double object construction.
When the comprehension or the formulation constraint of a conditional unit is empty, we write the empty set symbol $\emptyset$. This is the case here for the $\text{?ditransitive-clause}$ unit, which has only a formulation-constraint and no comprehension-constraint. This is typical for phrasal constructions, or other types of grammatical constructions, that add new meaning to a syntactic usage pattern, because the properties that the construction is looking for are syntactically expressed by properties of the conditional units and their ordering relations.

### 6.2 A formulation example

The double-object construction triggers in formulation when the predicate cause-receive and its arguments have to be expressed and when there are other units with the appropriate semantic characteristics as specified in the formulation-locks of the respective units on the right-hand side of the double object construction. Here is a concrete example, starting from the following meaning: There is a cake (obj-16), there is a bake-event (obj-6), with a baker (obj-17) and a baked object (obj-16). There is a cause-receive event (obj-6), with an entity causing this event (obj-17) who is a male person, an object that is being transferred (namely the cake, obj-16), and an entity receiving this object (obj-18) who is a female person. The root-unit is therefore:

<table>
<thead>
<tr>
<th>root</th>
</tr>
</thead>
<tbody>
<tr>
<td>referent: obj-6</td>
</tr>
<tr>
<td>predicates: {physobj(cake, obj-16), action(bake, obj-6), baker(obj-6, obj-17), baked(obj-6, obj-16), action(cause-receive, obj-6), causer(obj-6, obj-17), transferred(obj-6, obj-16), receiver(obj-6, obj-18), person(male, obj-17), person(female, obj-18)}</td>
</tr>
</tbody>
</table>

After the application of lexical constructions the transient structure contains the following units. There are no ordering constraints between them yet.
Now the double-object construction Con.5 can apply. The formulation-lock of the ?ditransitive-clause unit in this construction requires that the meaning includes:

\[
\text{cause-receive}(\text {?event}), \text{causer}(\text {?event}, \text {?causer}), \text{transferred}(\text {?event}, \text {?transferred}), \text{receiver}(\text {?event}, \text {?receiver})
\]

This is the case here, assuming the following binding-set:

\[
(\text {?event} . \text {obj-6})(\text {?causer} . \text {obj-17})(\text {?transferred} . \text {obj-16})
\]

\[
(\text {?receiver} . \text {obj-18}) (\text {?case-1} . \text {not-nominative})
\]

The right-hand side of this construction contains a unit for a ditransitive-clause which is now created:

\[
\text{ditransitive-clause-1}
\]

The formulation locks of the other units on the right-hand side of Con.5 match as well, giving the following bindings ?subj with he-unit-1, ?verb with bakes-unit-1, ?ind-obj with her-unit-1, and ?obj with cake-unit-1. So no new units have to be created but their constraints can be merged into the existing units in the transient structure. Variables are substituted for their bindings to construct the new transient structure after construction application. So we see new information in the verb-unit (bakes-unit-1)
because the variables ?subj-1 and ?dir-obj-1 have been replaced by their bindings: he-unit-1 and cake-unit-1 respectively, and for the noun phrase unit cake-unit-1 because ?case-1 is replaced by its binding, namely not-nominative.

(Ex.16)

Two things should be noted. (i) As mentioned earlier, units can receive extra information when the constraints in the conditional units are merged with the transient structure, but this merging process will block the application of the construction when incompatibilities are detected. For example, in “he bakes she (a) cake” “she” is nominative which clashes with the constraint on ?ind-obj that the case-value is not-nominative. (ii) Ex.15 and Ex.16 show that variables (here ?case-1 which is the case-value of case-unit-1) can appear in transient structures as well and they can get bound when a construction provides a concrete value for them (in this case not-nominative). This illustrates again the power of using logic variables. In effect, they cast a net of constraints over the transient structure and when a variable gets bound at some point, its value propagates in this net to all other occurrences.

Next we turn to the contributor. The constituents-feature of ditransitive-clause-1 are filled in with the different NPs and the verb and obj-17 has been added as receiver in the frame of the verb and the ind-obj in the syn-valence. These additions are highlighted in italics in the final transient structure:

<table>
<thead>
<tr>
<th>ditransitive-clause-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>predicates:</td>
</tr>
<tr>
<td>{action(cause-receive,obj-6),</td>
</tr>
<tr>
<td>causer(obj-6,obj-17),</td>
</tr>
<tr>
<td>transferred(obj-6,obj-16),</td>
</tr>
<tr>
<td>receiver(obj-6,obj-18)}</td>
</tr>
<tr>
<td>constituents:</td>
</tr>
<tr>
<td>{he-unit-1, verb-1,</td>
</tr>
<tr>
<td>her-unit-1, cake-unit-1}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>he-unit-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>referent: obj-17</td>
</tr>
<tr>
<td>sem-fun: referring</td>
</tr>
<tr>
<td>predicates: {person(male,obj-17)}</td>
</tr>
<tr>
<td>sem-cat: {physobj, animate}</td>
</tr>
<tr>
<td>phrasal-cat: NP</td>
</tr>
<tr>
<td>string: “he”</td>
</tr>
<tr>
<td>case: nominative</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>cake-unit-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>referent: obj-16</td>
</tr>
<tr>
<td>predicates: {physobj(cake,obj-16)}</td>
</tr>
<tr>
<td>sem-fun: referring</td>
</tr>
<tr>
<td>sem-cat: {physobj}</td>
</tr>
<tr>
<td>phrasal-cat: NP</td>
</tr>
<tr>
<td>string: “(a) cake”</td>
</tr>
<tr>
<td>case: not-nominative</td>
</tr>
</tbody>
</table>
From this transient structure, information about the form of the utterance gets extracted, yielding the following set of descriptions:

<table>
<thead>
<tr>
<th>String (he-unit-1, &quot;he&quot;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>String (bakes-unit-1, &quot;bakes&quot;)</td>
</tr>
<tr>
<td>String (her-unit-1, &quot;her&quot;)</td>
</tr>
<tr>
<td>String (cake-unit-1, &quot;(a) cake&quot;)</td>
</tr>
</tbody>
</table>

This gets translated by the renderer into the ordered set of strings: “he baked her (a) cake”.

### 6.3 A comprehension example

Here is a concrete example of comprehension, starting from the utterance “he baked her (a) cake”. After the application of the lexical constructions, we get the following initial transient structure:
The referents in all the NP units (such as ?obj-23 in cake-unit-2) are all variables, and the roles in the bake-predicate (the baker to be bound to ?obj-10, or the baked to be bound to ?obj-16) or its semantic valence (the actor ?obj-10 or the undergoer ?obj-16) are not related yet to these referents. Moreover there are several other variables not yet determined, such as the subject ?subject-2 or direct-object ?dir-object-2 in the syntactic valence frame of the verb, or the case-value ?case-2 of “(a) cake”. Notice also that there is no mention of an indirect object in the valence frame of ‘bake’, nor of a receiver in the predicates defining the predicates of the verb.

Now the comprehension-constraints of the conditional units on the right-hand side of the double-object construction are matched against this transient structure. This is successful with the following binding-list:

- (?subject-2 . he-unit-2) (?verb . bakes-unit-2) (?NP2 . her-unit-2)
- (?dir-object-2 . cake-unit-2) (?case-2 . not-nominative)

Variables in the predicates and in the frames are linked to the referents of the NP-units by the chain of bindings to the same variables: ?causer = ?obj-20 = ?obj-10, ?transferred = ?obj-23 = ?obj-16.

There are no comprehension-constraints for the the ditransitive-clause unit. This means that this unit can simply be created as part of the matching
phase of the construction, using the bindings already known. The unit
is called \textit{ditransitive-clause-2}. All variables bound to each other are
replaced by a single variable, as in Ex.20:

\begin{center}
\begin{tabular}{|c|}
\hline
ditransitive-clause-2 \\
\hline
\textbf{predicates:} \\
\{action(cause-receive,?obj-21), \\
causer(?obj-21,?obj-20), \\
transferred(?obj-21,?obj-23), \\
receiver(?obj-21,?obj-22)\}
\hline
\end{tabular}
\end{center}

(Ex.20)

Then merging takes place. First the formulation constraints are added
to each conditional unit. The units which are subject \textit{?subject-2} and
direct-object \textit{?dir-object-2}, can be substituted by their bindings, namely
\textit{he-unit-2} and \textit{cake-unit-2}, and \textit{?verb} is substituted by its binding \textit{bakes-unit-2}.
Second, information from the contributor is added, which includes additions
to the syntactic and semantic valence of the verb and the phrase structure.
We obtain the following final transient structure with the final changes high-lighted:
The constituent structure tree that is present in this feature structure is shown in Figure 3.

Figure 3: Constituent structure tree represented explicitly in the value of the constituents-feature of ditransitive-clause-2 in Ex.21.

7 Other grammatical perspectives

Fluid Construction Grammar integrates different views on the utterance, and we have already seen some of them:

- Phrase structure is realized using the feature constituents which stores the hierarchical relations between the different units, and the features lex-cat and phrasal-cat which contain the parts of speech or phrasal categories of each unit.

- Functional structure is realized both in terms of semantic functions and syntactic functions. Units have a sem-fun feature with values like predicating, referring, etc. These indicate how the predications contained in the meaning of the unit should be used to achieve communicative goals. The syn-fun feature has values like subject, determiner, modifier, etc. They are the syntactic reflections of the semantic functions.

- Argument structure is realized by the frame attached with words, such as 'bakes', that defines the possible semantic roles, by the semantic classes (sem-cat) that are associated with units, such as physobj or animate, and by defining the mapping between lexical meaning and argument structure in the construction.

To extend the ditransitive example so that it can deal with dependencies is straightforward. It is just necessary to add a feature to store the
dependents in the transient structure and to keep track of both the head and the dependents. This approach is shown in full detail in the web demonstration associated with this paper, as part of Example IV. Other linguistic perspectives that play a role in the double object construction can be added simply by adding additional features, for example a feature indicating the topicality or the complexity of the indirect object construction, which both co-determine whether the construction can apply.

8 Conclusions

This paper introduced some of the basic ideas of Fluid Construction Grammar and how they have been turned in operational data structures and algorithms. The explanation started from the notion of rewrite grammars, and focused then on a classical constructionist example where grammar adds meaning beyond what it is provided by the lexicon. There are many topics related to FCG that have not been discussed such as: How language processing can be fluid by accepting partial matches or coercing words or structures in roles they did not have yet, how language processing can cope with diversity by including scores with each construction, how processing can deal with a very large number of constructions both from the viewpoint of retrieving rapidly the most relevant construction (by hashing techniques) and of damping combinatorial search through the space of possible transient structures (by using heuristic search criteria). Another big topic is how grammatical constructions can be learned. FCG has been used with inductive statistical machine learning algorithms operating over corpora, although most of the work on FCG grammar learning so far has focused on using FCG to learn grammars in situated embodied interactions [Spranger et al.(2016)] using insight learning [Garcia-Casademont and Steels(2016)]. Another topic is how constructions can change in a population of language users. Much work has already been done in this area as well, some of it inspired by phenomena observed in actual language change [Steels(2012)], [Beuls and Steels(2013)]. At this moment FCG is sufficiently developed that all these topics can be researched through repeatable computational experiments.

9 Acknowledgement

The design and implementation of a computational platform for comprehension and formulation of language is a major undertaking that takes decades, both for design and for implementation, typically going through several iterations. Creating a community of users equally takes a long time. FCG is still in a development phase and many people have helped to get it to this point. Most of the activity has been taking place at the AI Laboratory of the University of Brussels (VUB), the Sony Computer Science Laboratory
in Paris, and more recently, the Complex Systems Group at the Institute for Evolutionary Biology (UPF-CSIC) in Barcelona.

The first implementations of components leading to FCG were started in the late nineties by Luc Steels, Angus McIntyre and Joris Van Looveren, later joined by Nicolas Neubauer. The first papers using FCG-like structures appeared in 2001 [?]. Around 2002, a new ambitious implementation effort started with Joachim De Beule as main developer of a new unification engine and Andreas Witzel as developer of the first web-based FCG interface. The FCG-engine became more stable, the first FCG workshop was held in Paris in September 2004, and more papers started to appear [Steels(2004b)], [Steels(2004a)].

Thanks to the EU-FP6 ECAgents project that started in January 2004, a new team of Ph.d. students could be assembled, which included Joris Bleys, Martin Loetzsch, Remi van Trijp and Pieter Wellens. This expansion led to many important innovations in terms of the representational power of the formalism, the processing engine and the general experimental environment. Thanks to the EU-FP7 ALEAR project, which started in 2008, a variety of case studies could be launched by an additional team of researchers, including Katrien Beuls, Nancy Chang, Sebastian Hoefer, Kateryna Gerasymova, and Vanessa Micelli. Significant progress was also made in parallel on embodied semantics, particularly by Wouter Van den Broeck, Martin Loetzsch, Simon Pauw, and Michael Spranger. This had in turn an important impact on grammar design, with major contributions, such as a new FCG-interface by Martin Loetzsch. This work was published in 2011-2012 [Steels(2011)] and the notation used is now referred to as FCG-2011.

More recently, Katrien Beuls, Paul Van Eecke and Remi van Trijp have been making important new contributions to the FCG-core by tackling well-known issues in grammar (such as long-distance dependencies or more flexible constituent ordering), which lead to further improvements in FCG itself. Michael Spranger has constructed a new object-oriented implementation, and additional case studies have been developed by a new generation of Ph.d students including Emilia Garcia Casademont, Miquel Cornudella, Yana King, Tânia Marques, and Paul Van Eecke. Paul van Eecke made major contributions to the new notation used in this paper and its mapping to the earlier FCG-2011 notation. The writing of the present paper was possible thanks to an ICREA fellowship to the author, and partially financed by the EU FP7 projects Insight (FET) and Atlantis (Chist-ERA).

References


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[Steels and Van Eecke(2017)] L. Steels and P. Van Eecke *Insight language learning with Pro- and Anti-Unification*. archiv


Appendix I: Glossary (alphabetic)

**Anti-unification:** Whereas unification tries to see whether and how one structure (e.g. a transient structure) can be made similar to another one (the lock or contributor of a construction schema), anti-unification does the opposite. It finds what is common and hence what is different between two structures. This is a fundamental operation in FCG to make construction application more fluid and for learning grammar but this is not further elaborated in this paper.

**(Atomic) symbol:** The primary building block of feature structures. They are either constants or variables. The meaning of a symbol does not come from its name but from its role in all structures where it appears.

**Comprehension-constraint:** Features and values of a unit that have to match in comprehension.

**Construction schema:** A construction schema (also called a construction) is an abstract schema that can be used to expand any aspect of a transient structure from any perspective and it can consult any aspect of the transient structure to decide how to do so. A construction associates a contributor (the left-hand side) with a lock or conditional part (the right-hand side).

**Construction-score:** Is a numerical value (between 0.0 and 1.0) that is associated with a construction. It reflects the success of the construction in past interactions and is used by the FCG-engine in case there is a competition between different constructions all matching with the same transient structure at the same step in the search space.

**Contributor:** Also called contributing part or left-hand side of a construction schema. It contains the set of constraints that are to be added to the transient structure, including new units if these units are not part of the transient structure yet.

**Feature:** A feature has a name and a value. Features represent properties of units and relations between units.
Feature-set: A set of features with their respective feature-names and values.

Feature structure: Datastructure for representing information about an utterance. It consists of a set of units with names and feature-sets.

Formulation-constraint: Features and values of a unit that have to match in formulation. They are also contributed in comprehension when the formulation lock is matching.

Lock: Also called conditional part or right-hand side (of a construction schema). It contains the set of constraints that have to be present in the transient structure in order for the construction schema to be applicable. The formulation lock is the set of constraints that are added and the comprehension lock the set of conditional units with their comprehension constraints.

Pathway: Also called linguistic pathway. A chain of transient structures obtained by consecutive application of constructions.

Transient structure: The feature structure built up during the comprehension or formulation of a concrete utterance.

Apply: The basic operation for applying a construction schema to a transient structure. It consists of two parts: a Match phase in which one lock of a construction is compared with the transient structure, and a Merge phase in which information from the contributor of the construction as well as from the other lock is added to the transient structure.

Match: The basic function for testing whether a transient structure fits with a construction lock. This operation is based on a variant of the unification operation, widely used in logically inference systems. It takes a transient structure, a lock, and an initial set of bindings as argument and then decides whether the transient structure fits in the lock, returning an extended set of bindings in case of success.

Merge: This operation takes a transient structure and the contributor of a construction and adds all information missing from the contributor to the transient structure. It uses a version of the unification algorithm to do this.

Unit: A unit has a name and a feature-set. It contains all information about a particular linguistic unit, e.g. a word, a morpheme, a phrase.

Value: The value of a feature can take the form of an atomic symbol, a formula (i.e. a set of predicates), a feature-set, or a set of values.

Variable: An atomic symbol which can get bound to another FCG-element (constants, sets, feature-sets, formulas, expressions). Variables are written as symbol of which the first character is ?, as in ?subject.

11 Appendix II: Definition of Match and Merge

This appendix defines the basic functions of FCG used in this paper with more precision. For a fully formal definition, the reader should consult
[Steels and De Beule(2006)]. Note that the following definition is not an algorithm but a formal specification which algorithms have to satisfy. In other words, it does not give an effective procedure for how the binding-sets are computed, only which binding-sets are needed. Of course, an effective algorithm has been used in the actual FCG implementation, which is similar to the algorithms used in the unification component of logic programming languages (such as [Martelli and Montanari(1982)]), but its exposition goes beyond the scope of this paper.

FCG conceptualizes a construction schema in terms of a contributor (written on the left-hand side of the arrow) and a lock or conditional part (written on the right-hand side). The feature-sets of the units on the right-hand side are split into two: a formulation-constraint and a comprehension-constraint. So a construction \( c \) is a triple:

\[
\begin{align*}
    c &= \langle \text{cont}_c, \text{prod}_c, \text{comp}_c \rangle \\
    \text{where cont}_c &\text{ is the set of units and their constraints forming the contributing part,} \\
    \text{prod}_c &\text{ is the set of conditional units with their formulation constraints} \\
    \text{and comp}_c &\text{ is the set of conditional units with their comprehension constraints. All of these are feature structures, i.e. lists of units with names and feature-sets.}
\end{align*}
\]

The core of construction application relies on two functions: Match and Merge, which are combined into Apply. In the following definitions, \( l \) stands for the lock and \( k \) stands for the key (the transient structure). Both of them are feature structures. \( m \) stands for the result which is true (T) or false (F) and \( b \) is used for the binding-set.

- **Match** checks whether a formulation or comprehension lock \( l \) matches with a transient structure \( k \) given a particular binding-set \( b \). It returns a truth-value \( m \in \{ T, F \} \):
  \[
  \text{Match}(l, k, b) \rightarrow m.
  \]

- **Merge** combines a transient structure \( k_{in} \) with \( p \), the information in a formulation or comprehension lock or the contributing part of a construction. It returns both a truth-value \( m \in \{ T, F \} \) if Merge is possible given a binding-set \( b \), and an expanded transient structure \( k_{out} \):
  \[
  \text{Merge}(p, k_{in}, b) \rightarrow m, k_{out}.
  \]

FCG uses logic variables and therefore the FCG-engine builds and utilizes a binding-set or substitution \( b \in V \times E \) where \( V \) is a set of variables and \( E \) their bindings, which could be any of the allowed elements of FCG: constants, variables, sets, formulas, and feature sets. Binding-sets are written as a set of dotted pairs. Because variables may be bound to variables, a binding-set can contain a chain, as in the following example:

\[
b = \{ (?x . ?y) (?y . ?z) (?x . ?u) (?z . singular) (?a . ?b) \}
\]
The grounded binding of a variable \( v \) is the non-variable binding of \( v \) in the chain, so the variables involved in the chain \(?x = ?y = ?z = ?u\), have all the element singular as their grounded binding. A variable may occur more than once in a binding-set, as is the case here for \(?x\) but there can be only one grounded-binding for each variable. Some variables, such as \(?a\) and \(?b\) in the above example, may remain unbound, i.e. they have no grounded binding.

We define the function \( \text{inst}(k, b) \rightarrow k' \) as the instantiation of a feature structure \( k \) given a binding-set \( b \). It is equal to a feature structure \( k' \) where all variables which have grounded-bindings are replaced by their grounded-binding, and all unbounded variables of the same chain are replaced by a single newly constructed variable, so that \( k' \) reflects that these variables are co-referential.

Apply\((c, k, b)\) with \( c \) a construction schema, \( k \) a transient structure, and \( b \) a set of bindings is defined as formulate\((c, k_{in}, b)\) in formulation and comprehend\((c, k_{in}, b)\) in comprehension.

1. Formulation: formulate\((c, k_{in}, b) \rightarrow k_{out}\)
   The formulation process first tries to match the formulation-lock prod\(_c\) with the transient structure \( k_{in} \), given a binding-set \( b \). If the match is successful, then the comprehension-lock comp\(_c\) is merged with \( k_{in} \) yielding \( k' \). If this was possible, then \( k' \) is merged with the contributing part cont\(_c\) to obtain \( k'' \). If this was possible as well, \( k_{out} = \text{inst}(k'', b) \). If any of these steps failed, the construction does not apply.

   More formally formulate is defined as follows:
   If Match\((\text{prod}_c, k_{in}, b) \rightarrow T\) then
   If Merge\((\text{comp}_c, k_{in}, b) \rightarrow T, k'\) then
   If Merge\((\text{cont}_c, k', b) \rightarrow T, k''\) then
   The new transient structure \( k_{out} = \text{inst}(k'', b) \).
   Otherwise \( c \) cannot be applied to \( k_{in} \) and Merge fails.

2. Comprehension: comprehend\((c, k_{in}, b) \rightarrow k_{out}\)
   Try to match the comprehension-lock comp\(_c\) with the transient structure \( k_{in} \), given a binding-set \( b \). If they match, then merge the formulation-lock prod\(_c\) with \( k \) yielding \( k' \). If this was possible, then merge \( k' \) with the contributing part cont\(_c\) to obtain \( k'' \). If this was possible as well, \( k_{out} = \text{inst}(k'', b) \). If any of these steps failed, the construction does not apply.

   More formally:
   If Match\((\text{comp}_c, k_{in}, b) \rightarrow T, b\) then
   If Merge\((\text{prod}_c, k, b) \rightarrow T, k'\) then
   If Merge\((\text{cont}_c, k', b) \rightarrow T, k''\) then
   The new transient structure is \( k_{out} = \text{inst}(k'', b) \).
   Otherwise \( c \) does not apply to \( k \).
I now define the Match and Merge functions.

**Matching**: \( \text{Match}(l, k, b) \rightarrow m \)

1. A feature structure \( l \) matches with a feature structure \( k \) iff there is a set of bindings \( b \) such that for every unit in \( l \) there is a corresponding unit in \( k \) that matches, i.e. \( \exists b (\forall u_l \in l, \exists u_k \in k \text{ Match}(u_l, u_k, b) \rightarrow T) \).

2. For a given binding-set \( b \), a unit \( u_l \) with name \( n_l \) and feature-set \( g_l \) matches with a unit \( u_k \) with name \( n_k \) and feature-set \( g_k \) iff the names of \( u_l \) and \( u_k \) are the same, i.e. \( n_l = n_k \), and both feature-sets match, i.e. \( \text{Match}(g_l, g_k, b) \rightarrow T \). If \( n_l \) or \( n_k \) are variables, then they must be part of the same chain in \( b \) or have the same grounded-binding.

3. For a given binding-set \( b \), a feature-set \( g_l \) matches with a feature-set \( g_k \) iff every feature \( f_l \in g_l \) matches with a corresponding feature \( f_k \in g_k \), i.e. \( \forall f_l \in g_l, \exists f_k \in g_k \text{ Match}(f_l, f_k, b) \rightarrow T \).

4. For a given binding-set \( b \), a feature \( f_l \) matches with a feature \( f_k \) iff the feature-names and values of these features match. In other words, given \( f_l = [f_{n_l} : v_l] \) and \( u_k = [f_{n_k} : v_k] \) then \( \text{Match}(f_l, f_k, b) \rightarrow T \) iff \( f_{n_l} = f_{n_k} \) and \( \text{Match}(v_l, v_k, b) \rightarrow T \).

5. To determine whether two values \( L = v_l \) and \( K = v_k \) match given a binding-set \( b \), we have the following cases:
   
   (a) \( L \) is an atomic constant symbol and \( K \) is also an atomic symbol, then they have to be equal, i.e. \( L = K \).

   (b) \( L \) is a variable, then either (i) \( K = L \), (ii) \( K \) is also a variable, in which case their grounded-bindings have to match or they have to be part of the same chain in \( b \), or (iii) \( K \) is not a variable (in other words a constant symbol, set, formula, or feature-set) in which case the grounded-binding of \( L \) in \( b \) has to match with \( K \).

   (c) \( L \) is a set. In that case, \( K \) has to be a set as well and all elements of \( L \) have to match with an element in \( K \): \( \forall e_l \in L, \exists e_k \in K \text{ where } \text{Match}(e_l, e_k, b) \rightarrow T \). Note that this implies a subset relation. The two sets do not have to have exactly the same elements.

   (d) \( L \) is a predicate with its arguments \( L = p_l(\arg_{1,l} ... \arg_{n,l}) \). In that case, (i) the value \( K \) has to be a predicate with arguments as well, i.e. \( K = p_k(\arg_{1,k} ... \arg_{n,k}) \), (ii) the predicates have to be equal, i.e. \( p_l = p_k \), and (iii) the arguments have to match, i.e. \( \forall \arg_{i,l} \in v_l, \exists \arg_{i,k} \in v_k \text{ where } \text{Match}(\arg_{i,l}, \arg_{i,k}, b) \rightarrow T \) and \( \forall \arg_{i,k} \in v_k, \exists \arg_{i,l} \in v_l \text{ where } \text{Match}(\arg_{i,l}, \arg_{i,k}, b) \rightarrow T \).

   (e) \( L \) is a feature-set. In that case, \( K \) has to be a feature-set as well and both feature-sets have to match, i.e. \( \text{Match}(L, K, b) \rightarrow T \).
In all other cases, \( \text{Match}(l, k, b) \rightarrow F \).

**Merging:** \( \text{Merge}(p, k, b) \rightarrow m, k_{\text{out}} \)

1. A feature structure \( p \) can Merge with a feature structure \( k_{\text{in}} \) iff \( \text{Match}(p, k_{\text{in}}, b) \rightarrow T \). The units in the merged feature structure \( k_{\text{out}} \) are either the result of merging the units of \( p \) with an existing unit in \( k \) or of adding a missing unit of \( p \) to \( k_{\text{out}} \). So, for every unit \( u_p \in p \) either there is a unit \( u'_k \in k_{\text{out}} \) such that \( u'_k \) is the result of merging \( u_p \) with its matching unit \( u_k \in k \), or, if no such merging was possible, \( u_p \) is additional to \( k_{\text{out}} \), i.e. \( u_p \in k_{\text{out}} \setminus k_{\text{in}} \).

2. Given a set of bindings \( b \), the merging of a unit \( p \) with name \( n_p \) and feature-set \( g_p \) with a unit \( k \) with name \( n_k \) and feature-set \( g_k \) is possible iff the names of \( p \) and \( k \) are the same, i.e. \( n_p = n_k \), and their feature-sets can be merged, i.e. \( \text{Merge}(g_p, g_k, b) \rightarrow T, g'_k \). If \( n_p \) or \( n_k \) are variables, then they must be part of the same chain or have the same grounded-binding. The new unit \( k' \) has as its name \( n_k \) if \( n_k \) is a constant, the grounded-binding of \( n_k \) if it is a variable, or a new variable which replaces all occurrences of \( n_p \) and \( n_k \). \( k' \) has as its feature-set \( g'_k \).

3. The merging of \( g_l \) with a feature-set \( g_k \) given a binding-set \( b \) is a new feature-set \( g'_k \) which is the union of all features \( f_l \in g_l \) that merged with a corresponding feature \( f_k \in g'_k \) and all features \( f_l \in g_l \) that could not be merged with a corresponding unit in \( g_k \).

4. Merging a feature \( f_l \) with a feature \( f_k \), given a binding-list \( b \), is possible iff \( \text{Match}(f_l, f_k, b) \rightarrow T \). Suppose \( f_l = [f_{n_l} : v_l] \) and \( f_k = [f_{k} : v_k] \) then the merged feature is \( f'_k = [f_{n_k} : v'_k] \) where \( v'_k \) is the result of merging the two values \( v_l \) and \( v_k \).

5. \( K' \) is the merge of two values \( P \) and \( K \), which are matching given a binding-set \( b \). We have the following cases:

   (a) If \( P, K \) are atomic constant symbols, then \( K' = K \). (We already know they are equal because otherwise the features of which they are values would not match.)

   (b) If \( P, K \) are variables, then a merge is possible iff \( \text{Match}(P, K, b) \rightarrow T \). \( K' = K \). (The variables form a chain in \( b \) so it does not matter which one we choose.)

   (c) If \( P \) and \( K \) are sets, then \( K' \) is equal to the set of all merged elements of \( P \) and \( K \) and those elements in \( P \) or \( K \) that did not match (and therefore cannot be merged):

\[
K' = \{ y' \mid x \in P \text{ and } \exists y \in K, \text{Merge}(x, y, b) \rightarrow T, y' \} \cup
\]
{\{x|\ x \in \ P \ \text{and} \ \neg \exists y \in \ K, \text{Match}(x, y, b) \rightarrow F\} \cup \{x|\ y \in \ K \ \text{and} \ \neg \exists x \in \ P, \text{Match}(x, y, b) \rightarrow F\}.

(d) If \(P, K\) are predicates with their arguments
\[P = K = \text{p}_p(\text{arg}_1, \ldots, \text{arg}_n).\]
Assuming that \(\text{Match}(P, K, b) \rightarrow T\), then \(K' = K\).

(e) If \(P, K\) are feature-sets then \(K'\) is equal to the union of all features that could be merged and the remaining features in \(P\) or \(K\) that did not match (and therefore could not be merged):
\[K' = \{y'|\ x \in \ P \ \text{and} \ \exists y \in \ K, \text{Merge}(x, y, b) \rightarrow T, y'\} \cup \{x|\ x \in \ P \ \text{and} \ \neg \exists y \in \ K, \text{Match}(x, y, b) \rightarrow F\} \cup \{x|\ y \in \ K \ \text{and} \ \neg \exists x \in \ P, \text{Match}(x, y, b) \rightarrow F\}.

In all other cases, \(\text{Merge}(p, k_{in}, b) \rightarrow F, nil\).